

Calculus III

Summer Review Packet

For problems that require written work, attach separate paper that is clearly numbered with the problems legibly worked out. This packet is due when school begins

Prerequisite Skills

A CALCULATOR MAY NOT BE USED ON THIS PART OF THE REVIEW

Directions: After examining the form of the choices, decide which is the best of the choices given and circle your choice.

Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

1. If $y = (2x^2 + 1)^4$, then $\frac{dy}{dx} =$

- A. $16x^3$ B. $4(2x^2 + 1)^3$ C. $4x(2x^2 + 1)^3$ D. $16(2x^2 + 1)^3$ E. $16x(2x^2 + 1)^3$

2. $\int x\sqrt{x^2 + 1} dx =$

- A. $\frac{x}{\sqrt{x^2 + 1}} + C$ B. $\frac{3}{4}(x^2 + 1)^{\frac{3}{2}} + C$ C. $\frac{1}{3}(x^2 + 1)^{\frac{3}{2}} + C$ D. $\frac{2}{3}(x^2 + 1)^{\frac{3}{2}} + C$ E. $\frac{1}{3}x^2(x^2 + 1)^{\frac{3}{2}} + C$

3. If $f(x) = x^3 - x + 3$ and if c is the only real number such that $f(c) = 0$, then c is between

- A. -2 and -1 B. -1 and 0 C. 0 and 1 D. 1 and 2 E. 2 and 3

4. A curve in the plane is defined parametrically by the equations $x = 2t + 3$ and $y = t^2 + 2t$. An equation of the line tangent to the curve at $t = 1$ is

- A. $y = 2x - 7$ B. $y = x - 2$ C. $y = 2x$ D. $y = 2x - 1$ E. $y = \frac{1}{2}x + \frac{1}{2}$

5. $\int_0^8 \frac{1}{\sqrt[3]{8-x}} dx$ is

- A. -6 B. 2 C. 6 D. 12 E. nonexistent

6. $\int x \sin x dx$

- A. $-\frac{1}{2}x^2 \cos x + C$ B. $-x \cos x + C$ C. $x \cos x - \sin x + C$
D. $-x \cos x + \sin x + C$ E. $-x \cos x - \sin x + C$

7. Let f be a differentiable function for all x . Which of the following must be true?

- I. $\frac{d}{dx} \int_0^3 f(x) dx = f(x)$
- II. $\int_3^x f'(x) dx = f(x)$
- III. $\frac{d}{dx} \int_3^x f(x) dx = f(x)$

A. II only B. III only C. I and II only D. II and III only E. I, II, and III

8. If $\sin(xy) = x^2$, then $\frac{dy}{dx} =$

A. $2x \sec(xy)$ B. $\frac{\sec(xy)}{x^2}$ C. $2x \sec(xy) - y$ D. $\frac{2x \sec(xy)}{y}$ E. $\frac{2x \sec(xy) - y}{x}$

9. For all x in the closed interval $[1,4]$, the function g is concave upwards. Which of the following tables could be the values of $g(x)$?

A.

x	$g(x)$
1	-10
2	-7
3	-6
4	-2

B.

x	$g(x)$
1	4
2	6
3	9
4	14

C.

x	$g(x)$
1	0
2	5
3	7
4	12

D.

x	$g(x)$
1	-4
2	-6
3	-8
4	-10

E.

x	$g(x)$
1	-2
2	-1
3	5
4	3

10. $\int \frac{dx}{x^2+4x} =$

A. $\int \frac{dx}{x} + \int \frac{dx}{x+4}$

B. $\int \frac{dx}{x^2} + \int \frac{dx}{4x}$

C. $\int \frac{dx}{x} - \int \frac{dx}{x+4}$

D. $\int \frac{dx}{4x} + \int \frac{dx}{4(x+4)}$

E. $\int \frac{dx}{4x} - \int \frac{dx}{4(x+4)}$

11. If $\int_0^4 (x^2 - 6x + 9)dx$ is approximated by 4 inscribed rectangles of equal width on the x-axis, then the approximation is

- A. 14 B. 10 C. 6 D. 5 E. 4

12. What is the 20th derivative of $y = \sin(2x)$?

- A. $-2^{20} \sin(2x)$ B. $2^{20} \sin(2x)$ C. $-2^{19} \cos(2x)$
D. $2^{20} \cos(2x)$ E. $2^{21} \cos(2x)$

13. What is the equation of the line tangent to the graph of $f(x) = 7x - x^2$ at the point where $f'(x) = 3$?

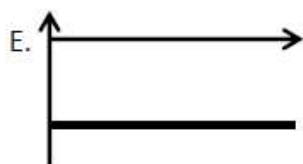
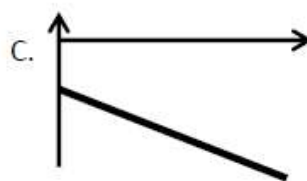
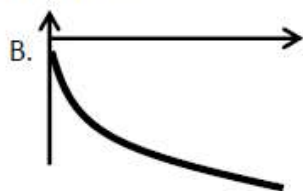
- A. $y = 5x - 10$ B. $y = 3x + 4$ C. $y = 3x + 8$ D. $y = 3x - 10$ E. $y = 3x - 16$

14. Suppose that $f(x)$ is a twice-differentiable function defined on the closed interval $[a,b]$. If $f'(c)=0$ for $a < c < b$, which of the following must be true?

- I. $f(a) = f(b)$
II. f has a relative extremum at $x=c$.
III. f has a point of inflection at $x=c$.

- A. None B. I only C. II only D. I and II only E. II and III only

15. A sky diver has a negative velocity while falling from an airplane. Before the sky diver opens the parachute, her velocity decreases quickly and then levels off due to air resistance. Which graph approximates the acceleration of the sky diver?



16. What are the first four nonzero terms in the power series expansion of e^{-4x} about $x = 0$?

- A. $1 + x + \frac{x^2}{2} + \frac{x^3}{3}$ B. $1 - 4x + 8x^2 - 32x^3$ C. $1 - 4x - 2x^2 - \frac{2}{3}x^3$
D. $1 - 4x + 8x^2 - \frac{32}{3}x^3$ E. $1 - 4x + 8x^2 - \frac{64}{3}x^3$

17. If f is a differentiable function such that the slope of the graph of f at each point $(x, f(x))$ is $\sqrt{x^2 - 2x}$, then the length of the graph of f between $(0, f(0))$ and $(2, f(2))$ is

- A. $\frac{1}{2}$ B. $\frac{2}{3}$ C. $\frac{3}{4}$ D. 1 E. 2

18. $\int_e^{e^2} \frac{dx}{x \ln x}$

- A. $\ln 2$ B. $\frac{1}{2}$ C. 1 D. 2 E. e

19. Let $f(x)$ be a continuous and differentiable function on the interval $0 < x < 1$, and let $g(x) = f(3x)$. The table below gives values of $f'(x)$, the derivative of $f(x)$. What is the value of $g'(0.1)$?

x	0.1	0.2	0.3	0.4	0.5	0.6
$f'(x)$	1.01	1.041	1.096	1.179	1.298	1.486

- A. 1.010 B. 1.096 C. 1.486 D. 3.030 E. 3.288

20. Which of the following integrals gives the total area of the region shared by both polar curves $r = 2 \cos \theta$ and $r = 2 \sin \theta$?

- A. $2 \int_0^{\frac{\pi}{4}} (\sin^2 \theta) d\theta$ B. $4 \int_0^{\frac{\pi}{4}} (\sin^2 \theta) d\theta$ C. $2 \int_0^{\frac{\pi}{2}} (\sin^2 \theta) d\theta$
D. $4 \int_0^{\frac{\pi}{4}} (\cos^2 \theta) d\theta$ E. $2 \int_0^{\frac{\pi}{4}} (\cos^2 \theta - \sin^2 \theta) d\theta$

21. $\lim_{h \rightarrow 0} \frac{2(x+h)^5 - 5(x+h)^3 - 2x^5 + 5x^3}{h}$ is

- A. 0 B. $10x^3 - 15x$ C. $10x^4 + 15x^2$ D. $10x^4 - 15x^2$ E. $-10x^4 + 15x^2$

22. If $\int_2^8 f(x)dx = -10$ and $\int_2^4 f(x)dx = 6$, then $\int_8^4 f(x)dx =$

- A. -16 B. -6 C. -4 D. 4 E. 16

23. If the graph of $y = x^3 + ax^2 + bx - 8$ has a point of inflection at $(2,0)$, what is the value of b ?

- A. 0 B. 4 C. 8 D. 12 E. 16

24. The position of a particle in the xy -plane is given by $x = 4t^2$ and $y = \sqrt{t}$. At $t = 4$, the acceleration vector is

- A. $(8, -\frac{1}{64})$ B. $(8, -\frac{1}{32})$ C. $(8, \frac{1}{32})$ D. $(32, -\frac{1}{32})$ E. $(32, \frac{1}{4})$

25. If f is a continuous function on the closed interval $[a,b]$, which of the following statements are NOT necessarily true?

- I. f has a minimum on $[a,b]$.
- II. f has a maximum on $[a,b]$.
- III. $f'(c) = 0$ for some number c , $a < c < b$

- A. I only B. II only C. III only D. I and II only E. I, II, and III

26. What are all the values of x for which the series $x - \frac{x^2}{2} + \frac{x^3}{3!} - \frac{x^4}{4!} + \dots + (-1)^{n+1} \frac{x^n}{n} + \dots$ converges?

A. $-1 \leq x \leq 1$

B. $-1 \leq x < 1$

C. $-1 < x \leq 1$

D. $-1 < x < 1$

E. All real numbers

27. $\sum_{n=0}^{\infty} \frac{(-1)^n (\pi)^{2n}}{(2n)!} =$

A. 1

B. -1

C. π

D. $\frac{\pi}{2}$

E. e^{π}

28. If $\frac{dy}{dx} = \frac{x}{y}$ and $y(3) = 4$, then

A. $x^2 - y^2 = -7$

B. $x^2 + y^2 = 5^2$

C. $x^2 - y^2 = 7$

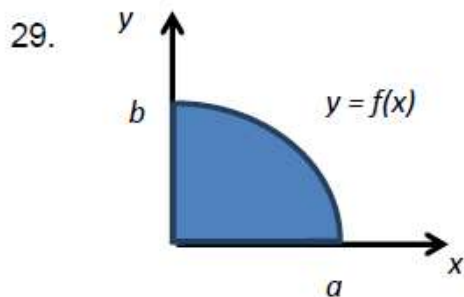
D. $y^2 - x^2 = 5$

E. $2x^2 - y^2 = 2$

A GRAPHING CALCULATOR IS REQUIRED FOR SOME QUESTIONS ON THIS PART OF THE REVIEW

Directions: After examining the form of the choices, decide which is the best of the choices given and circle your choice.

Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.



Let $f(x)$ be a continuous function and let A be the area of the shaded region in the figure above. Which of the following statements must be true?

I. $A = \int_0^a f(x) dx$

II. $A = \int_0^b f^{-1}(x) dx$

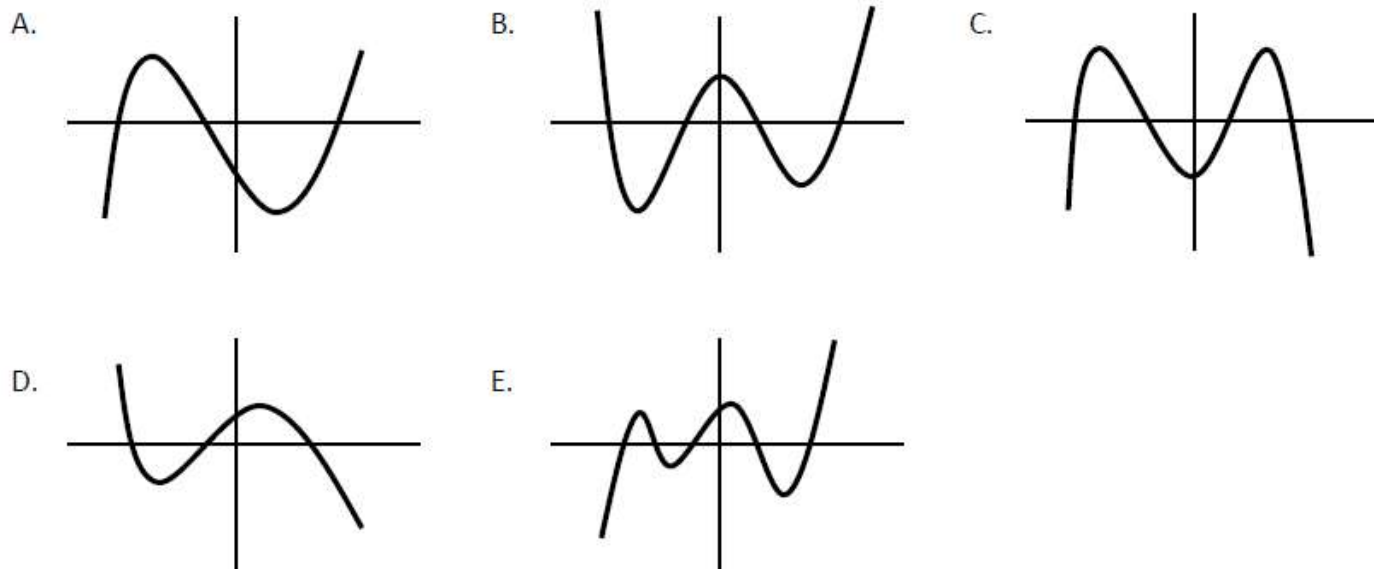
III. $A = \int_0^b f^{-1}(y) dy$

- A. I only B. II only C. III only D. I and II only E. I, II, and III

30. The Maclaurin series for a function f is given by $\sum_{n=0}^{\infty} \frac{x^n}{2^n}$. What is the value of $f^{(4)}(0)$, the fourth derivative of f at $x = 0$?

- A. 1 B. 2 C. 3 D. 4 E. 5

31. If $f'(x) = (x - a)(x - b)(x - c)$ and $a < b < c$, then which of the following could be the graph of $f(x)$?



32. If $f(3) = 7$ and $f'(x) = \frac{\sin(1+x^2)}{x^2-2x}$, then $f(5) \approx$

- A. -16.006 B. -9.006 C. -0.008 D. 6.992 E. 7.008

33. If $\sum_{n=1}^{\infty} |a_n|$ converges, then which of the following is true?

- I. $\sum_{n=1}^{\infty} a_n$ converges.
 II. $\sum_{n=1}^{\infty} a_n$ is absolutely convergent.
 III. $\sum_{n=1}^{\infty} -a_n$ converges.

- A. I only B. II only C. III only D. I and III only E. I, II, and III

34. The base of a solid is the region enclosed by the graph of $y = 3(x - 2)^2$ and the coordinate axes. If every cross section perpendicular to the x -axis is a square, then the volume of the solid is

- A. 8.0 B. 19.2 C. 24.0 D. 25.6 E. 57.6

35. When $x = \frac{\pi}{4}$, the rate at which $\sin^2 x$ is increasing is k times the rate at which x is increasing. What is the value of k ?

- A. $\sqrt{2}$ B. $\frac{\sqrt{2}}{2}$ C. 1 D. $\frac{1}{2}$ E. -1

36. Let f be a function whose seventh derivative is $f^{(7)}(x) = 10,000 \cos x$. If $x = 1$ is in the interval of convergence of the power series for this function, then the Taylor polynomial of degree six centered at $x = 0$ will approximate $f(1)$ with an error of not more than

- A. 2.45×10^{-5} B. 1.98×10^{-4} C. 3.21×10^{-2} D. 0.248 E. 1.984

37. If f is an antiderivative of $\frac{\tan^2 x}{x^2+1}$ such that $f(1) = \frac{1}{2}$, then $f(0) =$

- A. 0 B. 0.155 C. 0.345 D. 0.845 E. 1

38. Suppose that $f(x)$, $f'(x)$, and $f''(x)$ are continuous for all real numbers x , and that f has the following properties.

- I. f is negative on $(-\infty, 6)$ and positive on $(6, \infty)$.
- II. f is increasing on $(-\infty, 8)$ and decreasing on $(8, \infty)$.
- III. f is concave down on $(-\infty, 10)$ and concave up on $(10, \infty)$.

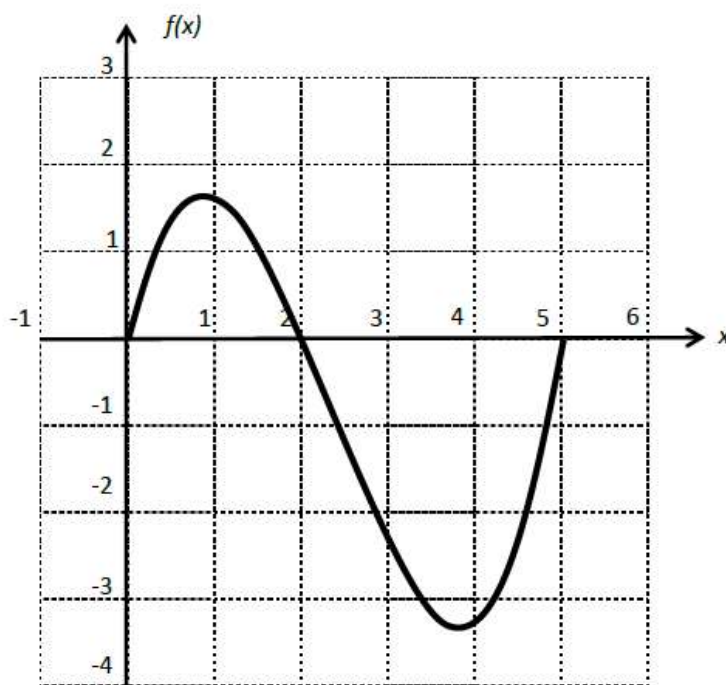
Of the following, which has the smallest numerical value?

- A. $f'(0)$ B. $f'(6)$ C. $f''(4)$ D. $f''(10)$ E. $f''(12)$

39. The present average price of a new car is \$14,500. The price of a new car is increasing at a rate of $120 + 180\sqrt{t}$ dollars per year. What will be the approximate average price of a new car five years from now?

- A. \$15,020 B. \$15,300 C. \$16,440 D. \$18,120 E. \$22,600

43.



The figure above shows the graph of a function $f(x)$ on the interval $[0,5]$. Which of the following definite integrals has the greatest value?

- A. $\int_0^1 f(x)dx$ B. $\int_0^2 f(x)dx$ C. $\int_0^3 f(x)dx$ D. $\int_0^4 f(x)dx$ E. $\int_0^5 f(x)dx$

44. The velocity vector of a particle moving in the xy -plane is $(3 - 4 \cos t, 4 \sin t)$ for all $t \geq 0$. When $t = 0$, the particle is at the point $(0, -1)$. Which statement best describes the motion of the particle?

- A. The particle moves around a circle.
 B. The particle moves along a sine graph.
 C. The particle moves to the left for all t .
 D. The particle moves to the right with a regular up and down motion.
 E. The particle moves generally to the right with a regular up and down motion, but periodically loops to the left.

45. The closed interval $[0, \pi]$ is partitioned into n equal subdivisions each of length $\Delta x = \frac{\pi}{n}$ by the numbers $x_0, x_1, x_2, \dots, x_{n-1}, x_n$ with $0 = x_0 < x_1 < x_2 \dots < x_{n-1} < x_n = \pi$.

$\lim_{n \rightarrow \infty} \sum_{i=1}^n x_i \cos(x_i) \Delta x$ is

- A. -2 B. -1 C. 1 D. 2 E. π